Ramp Behavior in Temperature Measurement

Intended vs. Actual Measurement

Unfortunately, it's not always possible to determine the temperature that you actually want to measure. Figure 1 gives an example of a measurement installation, where the sensor consists of an encapsulated thermocouple in an immersion pocket. The sensor could also be a Pt 100 sensor. We want to determine the temperature of the air that flows through the pipe. The air temperature in the pipe may vary over time, but the air velocity is constant. In the case that we will look at below, the air temperature in the pipe is significantly higher than the temperature in the pipe›s surroundings.

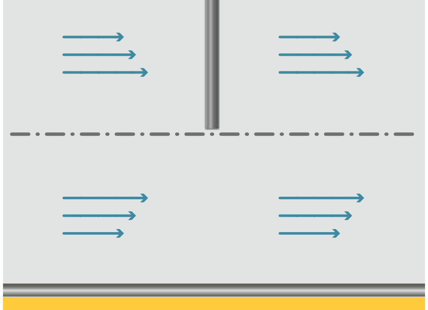

Figure 1: Instrumentation Setup.

The temperature we want to measure is constant

We want to consider the case where the air temperature in the pipe is constant, and we begin by studying the heat flow. Heat is transferred from the air in the pipe to the immersion pocket by forced convection. In the immersion pocket and the sensor, heat is transported through thermal conduction. Where the immersion pocket is attached to the pipe, heat is exchanged with the pipe wall through thermal conduction. A certain amount of heat exchange through thermal conduction can also occur with the pipe's insulation. Outside the insulation, heat is transported from the coupling head to the surroundings through convection and radiation. If the air velocity in the surroundings is negligible, the convective heat transport occurs through natural convection.

In this case, there is a small heat flow in the immersion pocket with the sensor, from the air in the pipe to the pipe's surroundings, the temperature of which is lower than the air temperature in the pipe. This means that the measuring point in the sensor will measure a temperature that is somewhat lower than the air temperature that we want to determine.

Thermocouple and Pt 100-type sensors are so-called contact sensors that measure their own temperature and absolutely nothing else. So in this case there is a difference between the temperature that we want to measure – the air temperature in the pipe – and the temperature that we actually measure – the temperature of the sensor. In some cases, the measurement error can be disregarded, but this must be decided on a case-by-case basis.

The temperature we want to measure varies with time

In the measurement installation shown in *Figure 1*, we now assume that the air temperature in the pipe varies with time. This almost always means that the response time to a temperature change in the pipe is of interest. When discussing response time, we often assume that the temperature we want to measure changes in the form of a step between two temperature levels. Unfortunately, these types of temperature changes rarely occur in engineering applications. What usually happens is that the temperature change between two levels takes the form of a ramp. We will therefore take a closer look at this case.

We now take the measurement installation in *Figure 1* as a basis and make the following assumptions. The inner diameter of the pipe is 200 mm, the outer diameter of the immersion pocket is 10 mm, and its length inside the pipe is 100 mm. Air flows in the pipe and the air temperature changes slowly at regular intervals between two levels, 30 °C and 180 °C. Each change takes approximately 20 minutes. We also assume that the heat flow from the immersion pocket with the sensor to the surroundings is negligible.

Figure 2 shows a basic picture of the temperature in the pipe and the measured temperature as a function of time. When the air temperature changes in the pipe, the sensor in the immersion pocket measures the air temperature with a certain lag. After some time, the temperature deviation becomes constant. Since, in this case, the heat flow from the immersion pocket to the pipe's surroundings is negligible, the measured temperature will connect with the constant upper temperature level after a certain length of time. The difference between the air temperature in the pipe and the measured temperature can be regarded as a measurement error. The time-dependent measurement error in this case depends, among other things, on the appearance of the ramp, the geometry and physical properties of the immersion pocket and sensor, and the heat transfer coefficient between the air in the pipe and the immersion pocket.

In some cases, we have a heat flow from the air in the pipe to the surroundings via the immersion pocket and the sensor. The measured temperature will then connect to a slightly lower temperature than the constant upper temperature level in the ramp.

To determine the difference between the temperature we want to measure and the temperature we actually measure, we can calculate the temperature distribution in the immersion pocket and the sensor. This is a three-dimensional time-dependent heat conduction problem. For the temperature, T, in °C in the immersion pocket with the sensor, T = T(t, x, y, z), where t is the time in seconds, and x, y and z are Cartesian coordinates in metres.

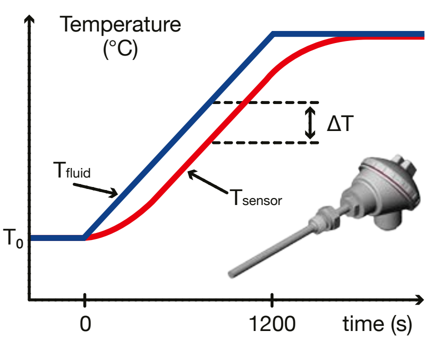

Figure 2: Temperatures as a function of time.

To calculate the temperature field, we must use the heat conduction equation with associated boundary conditions and initial condition. Unfortunately, there is no general analytical solution to this three-dimensional time-dependent problem, so we must use some suitable numerical method. In this case, it makes sense to use the Finite Element Method, FEM.

"Unfortunately, it's not always possible to determine the temperature that you actually want to measure."

If we ignore the heat flow in the immersion pocket and sensor to the surroundings in the axial direction (z-direction), we can simplify the problem. This restriction means, among other things, that we disregard the heat exchange with the pipe wall, the pipe insulation and the pipe's surroundings. With this assumption, we need only study what happens in a cross section of the immersion pocket and the sensor, which makes the problem two-dimensional, $T = T(t, x, y)$, where x and y are coordinates in the cross section. This problem is considerably simpler than the three-dimensional problem. Unfortunately, even in this case, a numerical method is almost always required to calculate the temperature field.

If the temperature difference within the cross-section of the immersion pocket and sensor were considerably smaller than the temperature difference between the surface of the immersion pocket and the flowing air in the pipe, the problem could be simplified further. If we disregard the temperature differences within the immersion pocket and sensor, the temperature would be $T = T(t)$. This means that the temperature within the immersion pocket and the sensor only depends on time t.

To solve the simplified problem, the so-called "lumped-heat-capacity method" can be used in some cases, which gives us a first-order differential equation. In many technically important cases, there is also an analytical solution to this problem.

To determine whether the "lumped method" is applicable, we can use a dimensionless number, the so-called Biot number, $Bi = (hL)/k$, where h is the heat transfer coefficient in $W/(m^2K)$ between the immersion pocket and the flowing air in the pipe, L is a characteristic length in metres for the geometry in question, and k is the thermal conductivity in W/(m K) in the immersion pocket and the sensor. If the immersion pocket is regarded as a long cylinder with perpendicular flow, the characteristic length is L = D/4, where D is the diameter of the immersion pocket in metres.

The Biot number is basically a measure of the ratio between the temperature difference within the cross section and the temperature difference between the surface of the immersion pocket and the flowing air in the pipe. The "lumped method" can be used if the Biot number is small. In engineering applications, the method generally gives acceptable results if Bi < 0.1.

An example of temperature change in the form of a ramp

To determine the magnitude of the measurement error, the simplest possible method should be used. In this case, an engineering assessment needs to be made. We therefore start by checking whether the "lumped method" is applicable.

The outer diameter of the immersion pocket is 10 mm, which gives the characteristic length L = 0.0025 m. The heat transfer coefficient varies along and around the immersion pocket, and we use a mean value. With an air velocity of 10 m/s in the pipe, the heat transfer coefficient is approximately 95 $W/(m^2K)$, if we regard the diving pocket as a long cylinder with a perpendicular flow. The physical data of the air varies with the temperature, and we should therefore use a mean temperature, which in this case is (30 + 180)/2 = 105 °C. If we assume that the immersion pocket and sensor mainly consist of stainless steel, k = 15 W/(m K).

The Biot number with these values is 0.015, and the "lumped method" can be used, since Bi < 0.1.

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When using the "lumped method" to calculate the change in temperature over time, we must always bear in mind that the method is approximate, and that the calculation is based on a number of conditions and assumptions. It is therefore very important to remember this when evaluating the calculation results.

If we now assume that the "lumped method" applies, we are able, with the current assumptions, to determine the change in sensor temperature over time based on the following differential equation

$$dT/dt + ((4h)/(\rho cD))T = ((4h)/(\rho cD))T_{Fluid}$$

where, ρ is the density of the immersion pocket in kg/m^3 and c its specific heat capacity in (Ws)/(kg K). Both the density and the specific heat capacity vary within the cross-section of the studied cylinder, which consists of the immersion pocket and the sensor. This means that we must use the mean values of both the density and the specific heat capacity.

The air temperature in the pipe in this case changes in the form of a ramp: $T_{Fluid} = T_0 + Bt$ where, T_0 is the air temperature at time t = 0, and B is a coefficient that characterizes the appearance of the ramp and is given in °C/second. The initial condition required for the solution to the equation in this case is $T = T_0$, i.e. the cylinder temperature is equal to the air temperature at time t = 0. We now assume that the parameters h, ρ, c and B can be regarded as constants.

With the introduced conditions, the differential equation has the analytical solution

$$T = T(t) = T_0 + Bt - (\rho cDB)/(4h) + ((\rho cDB)/(4h))\, e^{-(4ht)/(\rho cD)}$$

This solution applies as long as the air temperature in the pipe changes in the form of a ramp. In this case, the relationship applies during the time 0 < t < 1 200 seconds.

The first two terms in the equation solution are the temperature of the ramp, i.e. the temperature change of the air in the pipe. The last term in the equation solution represents the settling process, which starts at time t = 0. The term contains the expression $e^{-(4ht)/(\rho cD)}$, which decreases with time t. This means that the settling process will "die out" after some time – the settling time.

The penultimate term in the equation solution, (ρcDB)/(4h), is the constant deviation, ΔT °C, between the temperature of the air in the pipe and the measured temperature, which is obtained when the settling process has "died out"; $\Delta T = (\rho cDB)/(4h)$. *See also Figure 2.*

Based on the expression for ΔT, which represents the difference between the air temperature in the pipe and the temperature we measure, we can make a number of interesting observations. For the measurement error ΔT the following applies: $\Delta T = (\rho cDB)/(4h)$

If the air velocity in the pipe were to increase, the heat transfer coefficient h $W/(m^2K)$ will increase, and this means that the deviation ΔT will decrease. We also find that the faster the air temperature in the pipe changes (larger B), the greater the deviation ΔT.

The deviation ΔT also increases with the outer diameter D of the immersion pocket, but here the relationship becomes a little more complicated, since an increase in the diameter D also affects the value of the heat transfer coefficient h $W/(m^2K)$. If the diameter of the immersion pocket were to increase from 10 mm to 12 mm, the heat transfer coefficient h $W/(m^2K)$ would decrease by approximately 7%. Overall, this means that the deviation ΔT increases by almost 22% when the outer diameter increases by 20%.

For the current case, **B = (180 – 30)/1200 = 0.125 °C/s** applies, and for the cylinder we use values for stainless steel; ρ = 7900 kg/m^3 and c = 480 (Ws)/(kg K). With these values, we get ΔT = 13 °C. The settling process takes just over 7 minutes. It should be pointed out once again that the calculation method is approximate and based on a number of assumptions and conditions. Nevertheless, the result gives a good idea of the measurement method and its limitations, as well as the parameters that affect the deviation ΔT. We could put it as follows: "The calculation is not perfect, but it is good enough in an engineering context."

If a more accurate calculation is required, it will be necessary to study the two or three-dimensional time-dependent problem, and use an appropriate numerical method.

Some comments on the calculation results regarding the ramp measurement

The maximum deviation between the air temperature we want to measure and the sensor temperature we measure is in this case approximately 13 °C. This value is almost 9% of the difference between the two temperature levels 30 °C and 180 °C. If the primary interest in the temperature measurement concerns the two temperature levels in the ramp, we could, perhaps, accept the deviation ΔT = 13 °C, which only affects the part of the process where the temperature changes between the two levels. However, if you want to have control over the entire temperature process, a deviation of 13 °C is barely acceptable.

To reduce the measurement error that always arises when this type of equipment is used for measuring, you could, for example, use an immersion pocket with associated sensors that have a smaller outer diameter. If the outer diameter of the immersion pocket is 6 mm, the heat transfer coefficient will be approximately 120 $W/(m^2K)$, and the Biot number 0.012. We can therefore use the "lumped method". In this case, we get the deviation ΔT = 6 °C, which is approximately half the measurement error when the outer diameter of the immersion pocket was 10 mm. The settling time will also be about half as long.

You could also install a sensor that is specially designed to provide as little deviation as possible between fluid temperature and sensor temperature during dynamic processes. *See example in Figure 3.*

Figure 3: Example of a Pt100 sensor with reduced tip, model 7945000.

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