EQUATION FOR CALCULATING THE HEAT TRANSFER OF NATURAL CONVECTION

QUESTION ? ANSWER

Questions should be of general interest and be about temperature measurement techniques and/or heat transfer.

QUESTION: To estimate the measurement error of a surface sensor we need to calculate the heat flow from a horizontal pipe for hot water. The pipe is insulated with mineral wool and is situated inside our workshop. In a technical article I found the equation $Nu = 0.43(Gr\ Pr)^{0.25}$ which can be used to calculate the heat flow for natural convection. In a manual I found the same equation but with the coefficient 0.53 instead of 0.43. Which coefficient is correct?

Jan S

ANSWER: This question is very relevant and partly has to do with accuracy. The following discussion also applies to other equations of a similar type that are used to calculate heat flows. This particular equation can be used to calculate the natural convection for a very long pipe with a constant surface temperature and where the pipe is situated in a large space with stagnant gas having a constant temperature. Natural convection is also called free convection.

The equation is based on a combination of theory and experiment. Starting from the basic equations for heat transfer and flow, we can show theoretically that the dimensionless Nusselt number (Nu) for natural convection depends on two other dimensionless numbers: the Grashof number (Gr) and the Prandtl number (Pr). With the help of the Nusselt number we can then calculate the heat transfer coefficient. The dimensionless numbers are explained in greater detail at the end of this article. For many industrially relevant geometries such as pipes and plates, it has also been found that for rough calculations for natural convection, it is possible to use equations of the type

$$Nu = C(Gr\ Pr)^n$$

where C and n are coefficients, which depend on factors including the geometry in question and the product of two dimensionless numbers (Gr Pr).

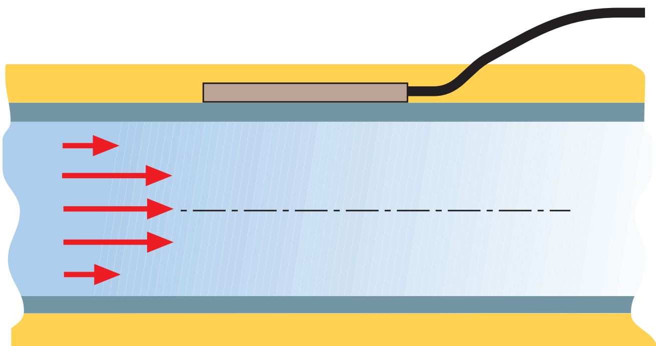
For a long horizontal pipe with a circular cross-section situated in a stagnant gas, we can find in most reference works the same value for the coefficient n: $n = 0.25$. In contrast, for the coefficient C we find that the value depends on which reference work we use; 0.41, 0.43, 0.47, 0.53 and 0.57 are some of the values we can find. The stated values apply on the condition that $(Gr\ Pr) < 10^9$. If you continue searching the literature you will definitely find additional values for the coefficient C. In this situation, it is really possible to wonder what value for the coefficient C is the correct one.

All the above-stated values for C are probably correct based on the conditions that applied when the coefficient was determined by experimentation. In addition to the conditions of a long horizontal pipe with a circular cross-section and a constant surface temperature, and situated in a stagnant gas having a constant temperature, there must also have been additional conditions governing the experiments. Examples of such other conditions might be how the pipe was mounted and which method was used to eliminate the effect of radiation when determining the constant C. What these other conditions were is almost never stated in the reference works. It may be possible to discover more information if you go to the original reports that document the experiments.

The wide variation of values of the coefficient C (in this case 0.41 – 0.57) says quite a lot about the accuracy you can expect when you calculate the heat transfer coefficient using the equation in question. You must also consider that the conditions governing your specific problem seldom match those that apply to the equation for natural convection that you are using. One such condition might be that your own pipe is situated close to a wall, and another one might be that the ambient air temperature is not constant and the air is not stagnant.

In conclusion, you must therefore always be aware of the large measurement uncertainty that exists when you use different equations to determine the convective heat flow.

When calculating the heat transfer from the pipe in question, in addition to including the convective heat transfer you must also include the heat transfer via radiation. In the room temperature range, heat transfer via radiation is often of the same order of magnitude as that via natural convection.



Dimensionless numbers for a horizontal pipe situated in a stagnant gas with the temperature T_{amb}

The Nusselt number, $Nu = hD/k$ where h is the heat transfer coefficient in $W/(m^2K)$, D the pipe's outer diameter in m and k the gas's thermal conductivity in W/(m K).

The Grashof number, $Gr = (g\ \beta\ \Delta T\ D^3)/\upsilon^2$, where $\beta = 1/T_{amb}$ is the gas's coefficient of volume expansion in 1/K, ΔT the temperature difference between the pipe's surface temperature and the gas temperature in K, D the pipe's external diameter in m and υ the gas's kinematic viscosity in m^2/s.

The Prandtl number, $Pr = (\upsilon\ \rho\ c_p)/k$ where υ is the gas's kinematic viscosity in m^2/s, ρ the gas's density in kg/m^3, c_p the gas's specific heat capacity in (Ws)/(kg K) and k the gas's thermal conductivity in W/(m K).

The quantities k, υ and ρ depend on the temperature and should be determined at a temperature that is the average of the pipe's surface temperature and the gas's temperature.

If you have questions or comments, contact Professor Emeritus Dan Loyd, LiU, dan.loyd@liu.se