TIME CONSTANT

QUESTION: I have come across the time constant τ in various contexts. What does it mean and how can it be used?

Johan H

ANSWER: The time constant is a measurement of how fast a parameter in a system changes over time. In an electrical system the voltage can be one such parameter and in a thermodynamic system the temperature can be one. Originally the prerequisite for the concept of the time constant was that the system's change over time must be governed by a first-order ordinary differential equation, but the concept is now also used in other contexts. An RC circuit, where a capacitor C discharges via an electrical resistance R, is one example of a system in which the voltage's variation over time follows a first-order differential equation. If the voltage was originally V_0 volt then the voltage, V volt, changes during the discharge in accordance with the equation

$$V = V_0\, e^{-t/(RC)}$$

where t is the time in seconds, R the resistance in ohms, and C the capacitor's capacitance in farad. The time required for the voltage to drop from V_0 to $V_0 e^{-1} = 0.37\, V_0$ (i.e. 37% of V_0) is called the time constant τ and is normally given in seconds. In this case $\tau = RC$.

If we overlook the temperature difference in a sheathed thermocouple that is sitting in a measuring circuit and regard the thermocouple as a "lump" with the temperature T(t) then the cooling is governed by a first-order ordinary differential equation. If the temperature of the thermocouple's surroundings is changed stepwise from the initial temperature T_0 °C to the ambient temperature T_{amb} °C ($T_{amb} < T_0$) then the thermocouple's temperature T(t) °C is changed in accordance with the equation

$$(T(t) - T_{amb})/(T_0 - T_{amb}) = e^{-t((hA)/(c\rho V))}$$

where h is the heat transfer coefficient in $W/(m^2K)$, A the thermocouple's heat transferring area in m^2, t the time in s, ρ the density in kg/m^3, c the specific heat capacity in (Ws)/(kgK) and V the volume in m^3. The capacitor's capacitance C in the RC circuit is represented by (ρcV) and the resistance R by 1/(hA). Because the thermocouple contains various materials, the quantities ρ and c are mean values. In the case of the thermocouple the time constant τ becomes

$$\tau = (c\rho V)/(hA).$$

The quantity ($c\rho V$), where (ρV) is the thermocouple mass, is a measurement of the thermocouple's ability to store energy. The greater the specific heat capacity and the greater the mass, the greater is the thermocouple's storage capacity – the time constant increases. The quantity 1/(hA) is a measurement of the thermal resistance. The smaller the value of the product (hA), the greater the thermal resistance, and it thereby takes longer time to change the thermocouple's temperature – the time constant increases.

However, there is an important difference between the RC circuit and the thermocouple. The RC circuit always has a constant time constant, $\tau = RC$, whether it is connected to an electrical network or is "lying on the shelf". In contrast, the thermocouple's time constant varies and depends on the heat flow to or from the thermocouple, which here is characterised by the quantity (hA), i.e. the product of the heat transfer coefficient and the heat transferring area.

QUESTION ? ANSWER

Questions should be of general interest and be about temperature measurement techniques and/or heat transfer.

If you have questions or comments, contact Professor Dan Loyd, LiU, dan.loyd@liu.se