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Measurement error during ramping

by Professor Dan Loyd

QUESTION: We are measuring the temperature in a tube with the aid of a Pt100 sensor, which sits inside a thermowell. The tube's inner diameter is 200 mm. The thermowell's outer diameter is 10 mm and the insert length of it is 100 mm. Contaminated air flows through the tube and the air temperature slowly varies at regular intervals between two levels: 30 °C and 180 °C. Each ramping cycle takes about 20 minutes. When we measure without using a thermowell, but with only the measurement insert (outer diameter: 6 mm) we get the same temperature as with the thermowell at the two levels, but a deviation when the air temperature changes. Is this a measurement error or is there another explanation?

Andreas B

ANSWER: In this case, a measurement error is less likely. The phenomenon you describe is namely an example of the temperature difference that can occur when we measure a temperature that changes in the form of a ramp – see diagram 1. When the air temperature changes, the sensor in the thermowell measures the temperature with some delay. The temperature deviation subsequently remains constant for a period of time. If the heat flow from the thermowell to the tube's surroundings is negligible, the measured temperature will reach the constant temperature level. The deviation, ΔT, depends on such factors as the geometry of the thermowell and the Pt100 sensor, their physical properties, and the heat transfer coefficient between the air and the thermowell. The deviation increases with the thermowell's outer diameter and decreases when the heat transfer coefficient increases.

When we measure using only the comparatively thin sensor (D = 6 mm), ΔT is less than when the sensor is in the thermowell (D = 10 mm). The heat transfer coefficient of the thin measurement insert is also greater than that of the thermowell, which further reduces ΔT. When doing both measurements there is for a period of time a constant deviation ΔT, but this is not the same in the two cases. Factors on which this comparison is based include the fact that there is a similar flow around both the measurement insert and the thermowell.

If the thermowell in question is of the standard type ("DIN, form B") the constant deviation can be estimated at 13 °C, if the air velocity is 10 m/s. The importance of the deviation must be assessed from case to case.

Extended article:

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General comments on calculating temperature deviations
We can calculate the temperature deviation by studying the temperature inside the thermowell and the Pt100 sensor, which is a three-dimensional unsteady-state thermal conduction problem. The temperature T in ^{o}C is a function of time and space, T = T(t, x, y, z), where t is the time in seconds, and x, y and z are Cartesian coordinates in metres. For the calculation we use the thermal conduction equation with the associated initial and boundary conditions. Unfortunately there is no general analytical solution to this three-dimensional problem; instead, we must use an appropriate numerical method. In this case the finite element method (FEM) works well.

The measurements with two different sensor installations give the same value for the constant temperature levels. This indicates that the heat flow in the axial direction (the z direction) in the thermowell and sensor and then to/from the tube and its surroundings is negligible. If the axial heat flow in the thermowell and the sensor can be disregarded, it is sufficient to study a cross-section, which makes the problem two-dimensional, T = T(t, x, y), and thereby simpler than the three-dimensional problem. Unfortunately, even in the two-dimensional case, a numerical solution to the problem is necessary in this case.

If the temperature difference inside the cross-section of the thermowell and the sensor is considerably less than the temperature difference between the surface of the thermowell and the air flow in the tube, the problem can be further simplified. If we disregard the temperature differences inside the thermowell and the sensor, then temperature is a function of the time only, T = T(t). To solve the problem, we can use what is called the "lumped-heat-capacity method", which then gives us a first order differential equation. In many technically important cases there is also an analytical solution to the problem.

Lumped-heat-capacity method
To determine if this method is suitable, we can use the Biot number, Bi = hL/k, where h [W/(m^2 K)] is the heat transfer coefficient between the thermowell and the air flow in the tube, L is the characteristic length in metres and k [W/(m K)] is the thermal conductivity inside the thermowell and the sensor. For a long cylinder, L = D/4, where D is the cylinder diameter in metres. The Biot number is a measurement of the relationship between the temperature difference inside the cross-section and the temperature difference between the surface of the thermowell and the air flow in the tube. The lumped-heat-capacity

method can be used if the Biot number is small. As a rule, for Bi < 0.1 the method gives acceptable results.

The thermowell's outer diameter is 10 mm, which gives us L = 0.0025 m. With an air velocity of 10 m/s the heat transfer coefficient is 95 W/(m^2 K) if we regard the thermowell as being a long cylinder. To determine the physical data of the air, we should use the mean temperature (30 + 180)/2 = 105 ^{o}C. If we assume that the thermowell and the sensor consist primarily of stainless steel, then k = 15 W/(m K). The Biot number is then 0.015 and we can use the lumped-heat-capacity method. However, always be aware that this method is approximative and is based on a number of assumptions.

Calculating the sensor temperature

The thermowell is regarded as a long cylinder. If we assume that the lumped-heat-capacity method can be used, then given the assumptions stated above, we can determine the sensor temperature with the differential equation

$$dT/dt + (4h/(\rho cD))T = (4h/(\rho cD))T_{Fluid}$$

where ρ is the density in kg/m^3 and c is the specific heat capacity in J/(kg K). Both the density and the specific heat capacity vary inside the cylinder, which consists of the thermowell and the sensor. This means that we must use mean values. The air temperature inside the tube varies in the form of a ramp

$$T_{fluid} = T_0 + Bt$$

where T_0 is the air temperature at the time t = 0 and B is a coefficient that characterises the ramp and is stated in ^{o}C/second. The initial condition required to solve the equation is $T = T_0$, i.e. the cylinder temperature is equal to the air temperature at the time t = 0.

With these conditions introduced, the differential equation has the analytical solution

$$T = T_0 + Bt - (\rho cDB)/(4h) + (\rho cDB)/(4h)e^{-(4ht)/(\rho cD)}$$

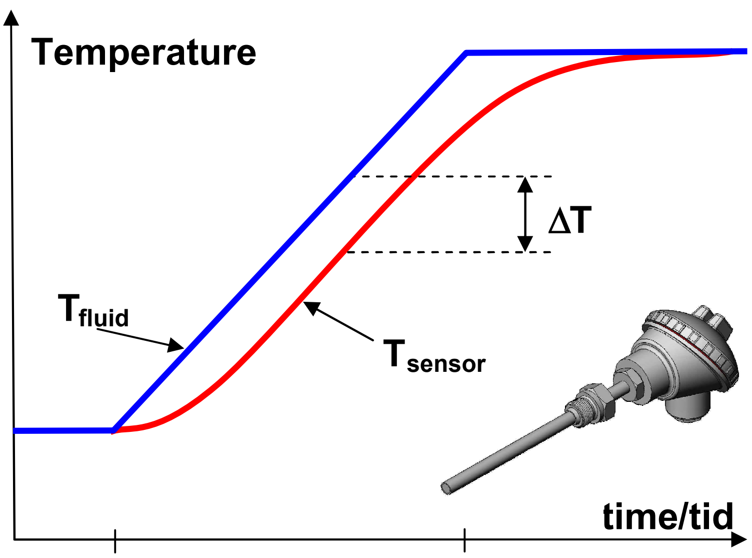

Diagram 1
The deviation ΔT increases with the thermowell's outer diameter D and decreases with the heat transfer coefficient h. The faster the air temperature changes (the larger the B value) the greater is the deviation ΔT.

Tel. +46 (0)490 25 85 00 Fax +46 (0)490 237 66
Pentronic AB, SE-590 93 Gunnebo, Sweden.
www.pentronic.se info@pentronic.se

This solution is valid as long as the air temperature inside the tube varies in the form of a ramp. In this case $0 < t < 1200$ seconds.

The last term in the solution represents the initial transient state that starts at the time $t = 0$. The second-last term is the constant deviation ΔT, which we get after the transient state. See further Diagram 1.

$$\Delta T = (\rho c D B)/(4h)$$

We can note that the deviation ΔT increases with the thermowell's outer diameter D and decreases with the heat transfer coefficient h. The faster the air temperature changes (the larger the B value) the greater is the deviation ΔT.

Determining the sensor temperature
In this particular case, $B = (180 – 30)/1200 = 0.125$ $^{o}C/s$. For the cylinder, we use the values for stainless steel: $\rho = 7900$ kg/m^3 and $c = 480$ J/(kg K). With these values, we get $\Delta T = 13$ ^{o}C. The initial transient state takes just over 7 minutes. It should be noted that the calculation method is approximative and that it is based on a number of assumptions. However, the result still provides good information about the measurement method and the parameters that influence the deviation ΔT. If a more precise calculation is desired, then you must study the two- or three-dimensional problem and use a suitable numerical method.

Acceptable temperature deviation
In this case, the maximum deviation between the air temperature and the sensor temperature is about 13 ^{o}C. This value is almost 9% of the difference between the two temperature levels of 30 ^{o}C and 180 ^{o}C. If the primary reason for measuring the temperature is the two temperature levels, it may be possible to accept the deviation, which only affects that part of the sequence when the temperature varies between the two levels.

If, however, the aim is to measure the entire temperature sequence then a deviation of 13 ^{o}C is scarcely acceptable. To reduce the deviation that always occurs when measuring with this method, we can, for instance, use a thermowell with a smaller outer diameter. It is also possible to install a sensor that is specially designed to give as little deviation as possible between the fluid temperature and the sensor temperature in dynamic processes.

If you have comments or questions, contact Professor Dan Loyd at the Institute of Technology at Linköping University: dan.loyd@liu.se