Calculating response time for different conditions?

Data sheets often give a response time or time constant for conditions that are more or less clear. Is it possible to easily calculate new response times for changed conditions? In this article Professor Dan Loyd explains the theory. We will supply practical examples at a later date.

To determine the new response time, in many cases it is necessary to take new measurements. Using a number of simplifications we can sometimes approximately estimate the new response time based on the previous measurements. The sensor temperature is affected by factors like the flow around the sensor, the sensor's physical design, and the mounting attaching it to the pipe wall; see Figure 1. The temperature inside the sensor varies according to both time and position. What we want to measure is the fluid temperature, T_{fluid}, but the system displays the temperature at the measuring point with some time lag.

If we (a) disregard the heat exchange with the mounting that attaches the thermocouple to the pipe wall and (b) assume that the temperature inside the sensor, T °C, depends solely on the time (t) in seconds, we can regard the installation as a first order measurement system. The sensor's temperature, T(t), is then governed by the differential equation

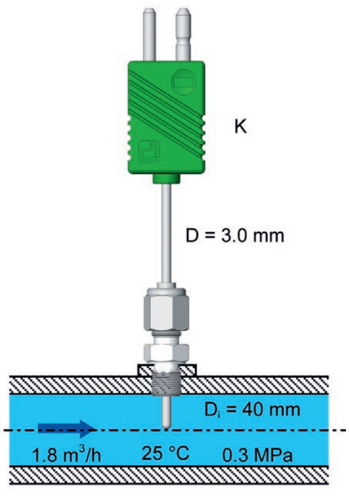

Figure 1. Temperature measurement in a pipe using a type K thermocouple with an insulated measuring point. The response time has been measured at t_{50} = 0.51 s. After a rebuild, the conditions have changed: Flow 0.9 m³/h, water temperature 40 °C and pressure 0.2 MPa. Can we easily calculate the new response time?

$$dT/dt + (\alpha A/\rho V c_p) T = (\alpha A/\rho V c_p) T_{fluid}$$

where α is the heat transfer coefficient in W/m² K, A the sensor's heat transferring surface in m², V the volume in m³, ρ the density in kg/m³ and c_p the specific heat capacity in J/kg K. The fluid's temperature, T_{fluid}, normally varies with time.

Our assumptions mean that we must use the mean value of the heat transfer coefficient and of the density and specific heat capacity of the sensor. Assumption (b) about the sensor temperature, T(t), requires that the temperature difference within the sensor must be small relative to the temperature difference between the fluid and the surface of the sensor, when a temperature change occurs in the fluid.

TIME CONSTANT AND RESPONSE TIME

Based on our assumptions, we can now calculate the measurement system's time constant, $\tau = (\rho V c_p)/(\alpha A)$.With a step-wise temperature change, the time constant is the time it takes for a (first order) system to achieve 63% of the temperature change, which corresponds to (1 – 1/e) of the change.

For a given installation, the time constant depends on the value of the heat transfer coefficient. The lower the value of the coefficient, the longer the time constant. An unused sensor on the warehouse shelf has no time constant; in contrast, an installed sensor with a specific heat transfer coefficient can have a time constant.

Figure 2 presents the difference between a theoretical and a real measurement system. The difference is caused in part by assumptions (a) and (b) for the equation (1).

CHANGED CONDITIONS

In the time constant expression, the heat transfer coefficient is only affected when the flow in the pipe changes (see Figure 1). In its turn, the heat transfer coefficient depends on both the flow velocity and the fluid temperature. The effect of the pressure on the heat transfer coefficient is in most cases negligible for fluids. The temperature's effect on the sensor's density, specific heat capacity and geometry is also completely negligible.

The thermocouple can be approximated to a very long cylinder through which a flow travels at a constant velocity normal to the cylinder. The literature gives a number of different expressions of the heat transfer coefficient for this case of flow. Depending on which relationships we use, we will get various values for the coefficient.

The time constant of the original case of flow is assumed to be τ seconds. After the changes, the heat transfer coefficient falls from 7300 W/m² K to 5600 W/m² K and the time constant increases to τ 7300/5600. If we assume that the response time increases in the same way as the time constant, the new response time will be approximately 0.7 seconds.

However, this estimate is based on a large number of assumptions, and the resulting estimate must therefore be used with great care. Assumption (b) is in this case a large approximation. To be on the safe side, you should measure the response time after the changes.

CHANGING THE FLUID

If in this case we replaced the water with another fluid, such as air, we can use the same technique as that described above to estimate the new response time. As before, the conditions are that we must be able to regard both the original and the new measurement equipment as first order measurement systems. This usually applies in most cases when changing from a fluid to a gas.

The opposite situation – from gas to fluid – is more complex and requires a careful check that the new fluid-based system can be regarded as a first order measurement system.

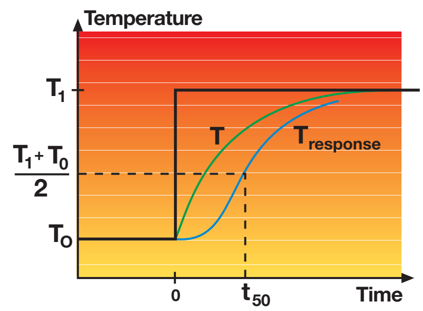

Figure 2. The temperature T for a first order measurement system and $T_{response}$ time for a real system with a step-wise change to the fluid temperature.

If you have comments or questions, contact Professor Dan Loyd at the Institute of Technology at Linköping University: dan.loyd@liu.see